Theory of spin accumulation and spin-transfer torque in a magnetic domain wall

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We studied the spin accumulation and spin-transfer torque in a magnetic domain wall by solving the Boltzmann equation for spin accumulation with the diffusion approximation. We obtained analytical expressions of spin accumulation and spin-transfer torque. Both the adiabatic and the nonadiabatic components of the spin-transfer torque oscillate with the thickness of the domain wall. We showed that the oscillating component plays a dominant role in the nonadiabatic torque when the domain wall is thinner than the spin-flip length. We also showed that the magnitude of the nonadiabatic torque is inversely proportional to the thickness of the domain wall.

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Spin-dependent electron transport in magnetic nanostructures results in many interesting phenomena such as the giant magnetoresistance effect[1] and current-induced magnetization dynamics.[2,3] Recently, such spin-dependent phenomena in magnetic domain walls have been investigated due to great interest in their potential application to spin-electronics devices such as spin-motive-force memory[4,5] and racetrack memory.[6] In these devices, data are stored by moving the domain wall using spin-transfer torque.

In 2004, Zhang and Li showed that spin-transfer torque in a domain wall can be decomposed into two parts, the adiabatic and nonadiabatic torque.[7] The adiabatic torque lies along the spatial gradient of the local magnetization while the nonadiabatic torque is perpendicular to this direction. Assuming that the spin accumulation obeys the phenomenological diffusion equation and is spatially independent of the domain wall, Zhang and Li[7] showed that the ratio of the magnitudes of the adiabatic and nonadiabatic torques is determined by the precession frequency of the spin accumulation due to the exchange coupling and the spin-flip scattering time, and that the nonadiabatic torque is about two orders of magnitude smaller than the adiabatic torque.

The thickness of a domain wall is determined by the competition of the exchange coupling between the localized magnetizations and the magnetic anisotropy, and is usually on the order of 100 nm for conventional ferromagnetic metals such as Fe, Co, Ni, and their alloys. However, recent developments in the processing technology for nanostructures have allowed the production of a domain wall whose thickness is on the order of 1–10 nm, by reinforcing the shape anisotropy of a magnetic nanowire[8] or trapping a domain wall in a current-confined-path geometry.[9] These developments motivated us to study the transport phenomena in a thin domain wall.[10–12] For such a thin domain wall, we cannot assume the spatial independence of the spin accumulation. Thus, it is important to estimate spin-transfer torque by taking into account the spatial variation in the spin accumulation, which would be different from the estimation by Zhang and Li.[7] Recently, Vanhaverbeke and Viret[13] calculated spin-transfer torque in a thin domain wall by numerically solving the time-dependent Larmor equation of the spin accumulation in a moving frame and showed that the nonadiabatic torque is one order of magnitude larger than that

estimated by Zhang and Li[7] when the thickness of the domain wall is comparable to the Lamor precession length.

In this Brief Report, we study spin accumulation and spin-transfer torque in a domain wall by solving the Boltzmann equation with a diffusion approximation. We obtained the analytical expressions of spin accumulation and spin-transfer torque. Both the adiabatic and the nonadiabatic components of the spin-transfer torque oscillate with the thickness of the domain wall. We show that the oscillation plays a dominant role in the nonadiabatic torque when the domain-wall thickness is less than the spin-flip length, which is defined by the product of the Fermi velocity and the spin-flip scattering time. For a domain wall that is much thinner than the spin-flip length, the nonadiabatic torque is about one order of magnitude smaller than the adiabatic torque, which is one order of magnitude larger than that estimated by Zhang and Li[7] and qualitatively consistent with the results of Vanhaverbeke and Viret.[13] We also showed that the magnitude of the nonadiabatic torque is inversely proportional to the thickness of the domain wall.

We considered electron transport in a one-dimensional magnetic nanowire with a 180° domain wall which lies over $-d/2 \leq x \leq d/2$, where d is the thickness of the domain wall. We assume that the interaction between the conducting (s-like) electrons and the localized (d-like) electrons is described by an sd exchange interaction, $\hat{H}_{sd} = -(J/2)\hat{\boldsymbol{\sigma}} \cdot \hat{\mathbf{S}}$, where $\hat{\boldsymbol{\sigma}}$ is the vector of the Pauli matrices, J is the sd exchange coupling constant, and $\hat{\mathbf{S}}(x) = (0, -\sin\theta, \cos\theta)$ is the unit vector pointing along the direction of the localized spin angular momentum. The angle $\theta(x)$ is given by 0 for $x < -d/2$, $(\pi/d)(x+d/2)$ for $-d/2 < x < d/2$, and π for $x > d/2$, respectively.

Following Simanek and Rebei,[14,15] we employ the rotating frame where basic unit vectors are defined as $\mathbf{e}_x = \alpha^{-1}\hat{\mathbf{S}} \times (\partial\hat{\mathbf{S}}/\partial x)$, $\mathbf{e}_y = -\alpha^{-1}\partial\hat{\mathbf{S}}/\partial x$, and $\mathbf{e}_z = \hat{\mathbf{S}}$, respectively. We assume that the direction of the localized spin varies slowly compared to the Fermi wavelength λ_F, i.e., $\alpha = d\theta/dx \ll 2\pi/\lambda_F$; thus, we could neglect the higher-order terms of α in the following calculation. The spin accumulation and spin-transfer torque in a domain wall are obtained by solving the Boltzmann equation for the Wigner function defined as $\hat{f}(x,p_x) = [f(x,p_x)\hat{1} + \mathbf{g}(x,p_x) \cdot \hat{\boldsymbol{\sigma}}]/2$, where $f(x,p_x)$

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and $\mathbf{g}(x,p_x)$ represent the charge and spin-distribution functions, respectively. The spin accumulation $\mathbf{s}$ and the spin current density $\mathbf{j}$ are defined as

$$\mathbf{s} = \int \mathbf{g} \frac{d^3\mathbf{p}}{(2\pi\hbar)^3}, \tag{1}$$

$$\mathbf{j} = \int v_x \mathbf{g} \frac{d^3\mathbf{p}}{(2\pi\hbar)^3}, \tag{2}$$

respectively. It should be noted that the dimensions of $\mathbf{s}$ and $\mathbf{j}$ are density and density times velocity, respectively. The diffusion approximation, $\int v_x^2 \mathbf{g} d^3\mathbf{p}/(2\pi\hbar)^3 \simeq (v_F^2/3)\mathbf{s}$, is applied to the Boltzmann equation.[14] Up to the first order of α, the transverse components of the spin accumulation, s_x and s_y, and spin current, j_x and j_y, obey the following equations:

$$\frac{\partial s_x}{\partial x} = -\frac{1}{2D} j_x + \frac{\omega_J \widetilde{T}}{D} j_y, \tag{3}$$

$$\frac{\partial s_y}{\partial x} = -\frac{\omega_J \widetilde{T}}{D} j_x - \frac{1}{2D} j_y, \tag{4}$$

$$\frac{\partial j_x}{\partial x} - \omega_J s_y + \frac{2}{\tau_{sf}} s_x = 0, \tag{5}$$

$$\frac{\partial j_y}{\partial x} + \omega_J s_x + \frac{2}{\tau_{sf}} s_y = \alpha j_z, \tag{6}$$

where $\omega_J = J/\hbar$ is the Larmor precession frequency, $\widetilde{T}$ is the momentum relaxation time, τ_{sf} is the spin-flip scattering time and $D = v_F^2 \widetilde{T}/3$ is the diffusion constant.[14,15] The longitudinal spin current j_z in Eq. (6) is given by $j_z = \beta j_e/(-e)$, where β and j_e are the spin-polarization factor and the electric current density, respectively. In our definition, the positive electric current corresponds to the electron flow along the $-x$ direction.

The physics behind Eqs. (3)–(6) are as follows. Traveling through the domain wall, the conducting electrons vary the direction of their spin along the localized spin angular momentum $\hat{\mathbf{S}}$. Then, spin accumulation and spin current polarized along the y direction ($\propto \partial\hat{\mathbf{S}}/\partial x$) are induced [see Eq. (6)]. The accumulated spins precess around $\hat{\mathbf{S}}$ due to the sd exchange coupling with the precession frequency ω_J. Then, the x components of the spin accumulation and spin current are induced [see Eq. (5)]. Equations (3) and (4) relate the spin accumulation and spin current by the diffusion constant.

Before estimating the spin accumulation and spin-transfer torque in a domain wall, we should emphasize the validity of our calculations. Since Eqs. (3)–(6) are obtained by applying the diffusion approximation to the Boltzmann equation, they are valid for $d \gtrsim l_{mfp}$, where $l_{mfp} = v_F \widetilde{T}$ is the mean-free path of the conducting electrons. For a domain wall the thickness of which is much smaller than the mean-free path, i.e., $d \ll l_{mfp}$, the Boltzmann equation should be solved without the diffusion approximation. Moreover, in such a very thin domain wall, we cannot neglect the higher-order terms of α.
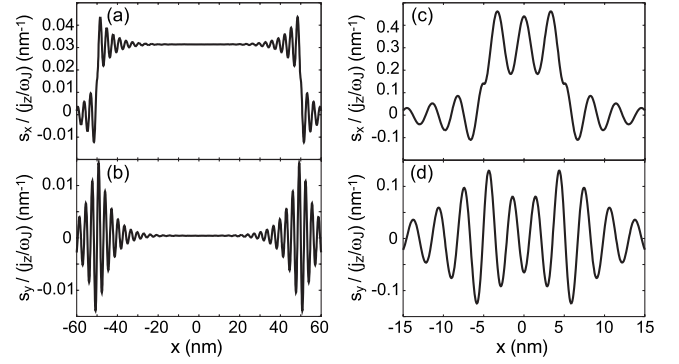

FIG. 1. The spatial variation in the transverse spin accumulations for a thick ($d=100$ nm) and thin ($d=10$ nm) domain wall; (a) s_x for $d=100$ nm, (b) s_y for $d=100$ nm, (c) s_x for $d=10$ nm, and (d) s_y for $d=10$ nm, respectively. The magnitudes of s_x and s_y are divided by j_z/ω_J [see Eq. (7)].

We assume that the transverse spin accumulation and spin current are such that they vanish at the limit of $|x| \to \infty$ and are continuous at $x = \pm d/2$. Then, solving Eqs. (3)–(6), the transverse spin accumulations in the domain wall are obtained as $s_x = \mathrm{Re}[s_+]$ and $s_y = \mathrm{Im}[s_+]$, where

$$s_+ = \frac{\pi(1+i\zeta)j_z}{\omega_J d(1+\zeta^2)}\left[1 - \exp\left(-\frac{d}{2\ell}\right)\cosh\left(\frac{x}{\ell}\right)\right]. \tag{7}$$

Here $\zeta = 2/(\omega_J \tau_{sf})$ and ℓ is given by

$$\frac{1}{\ell} = \sqrt{\frac{1}{2D}(1 + 2i\omega_J\widetilde{T})\left(i\omega_J + \frac{2}{\tau_{sf}}\right)}, \tag{8}$$

where $k_r = \mathrm{Re}[1/\ell]$ and $k_i = \mathrm{Im}[1/\ell]$ characterize the oscillation and damping of s_x and s_y due to the sd exchange coupling and the spin-dependent scattering, respectively. As shown in Eq. (7), the transverse spin accumulations can be decomposed into spatially independent (first) and dependent (second) parts.

Figure 1 shows the spatial dependence of the transverse spin accumulations, s_x and s_y, for thick ($d=100$ nm) and thin ($d=10$ nm) domain walls, respectively. For convenience, s_x and s_y are divided by j_z/ω_J [see Eq. (7)]. The parameters are taken to be $J=1.0$ eV, $\tau_{sf}=10^{-4}$ ns, and $l_{mfp}=3.0$ nm, respectively. The Fermi velocity is given by $v_F = \sqrt{2\varepsilon_F/m}$, where the Fermi energy is taken to be $\varepsilon_F = 5.0$ eV. These are typical values for the conventional transition ferromagnetic metals.[16] As shown in Figs. 1(a) and 1(b), for a thick domain wall, the spin accumulation in the domain wall is nearly spatially independent except at the boundaries of the domain wall $x = \pm d/2$. On the other hand, as shown in Figs. 1(c) and 1(d), for a thin domain wall, the spin accumulations vary in the domain wall and we cannot assume the spatial independence of the spin accumulations.

Let us estimate spin-transfer torque in the domain wall, which is defined as

$$\boldsymbol{\tau} = \int_{-d/2}^{d/2} \omega_J \mathbf{s} \times \hat{\mathbf{S}} dx. \tag{9}$$

The Landau-Lifshitz-Gilbert equation for the localized magnetization $\hat{\mathbf{M}} = -\hat{\mathbf{S}}$ with the torque $\boldsymbol{\tau}$ is given by

$$\frac{\partial \hat{\mathbf{M}}}{\partial t} = -\gamma \hat{\mathbf{M}} \times \mathbf{B} + \alpha_0 \hat{\mathbf{M}} \times \frac{\partial \hat{\mathbf{M}}}{\partial t}$$

$$+ \frac{\gamma \hbar}{2\pi M} \tau_y \frac{\partial \hat{\mathbf{M}}}{\partial x} + \frac{\gamma \hbar}{2\pi M} \tau_x \hat{\mathbf{M}} \times \frac{\partial \hat{\mathbf{M}}}{\partial x}, \tag{10}$$

where γ is the gyromagnetic ratio, $\mathbf{B}$ is the effective magnetic field, M is the magnitude of the magnetization and α_0 is the Gilbert-damping constant. $\tau_y = \mathbf{e}_y \cdot \boldsymbol{\tau}$ and $\tau_x = \mathbf{e}_x \cdot \boldsymbol{\tau}$ correspond to the adiabatic and nonadiabatic torque, respectively. By using Eq. (7), we find that

$$\tau_y = \frac{\pi \beta j_e}{e(1+\zeta^2)} - \frac{\pi \beta j_e[k_r - e^{-k_r d}(k_r \cos k_i d - k_i \sin k_i d)]}{ed(1+\zeta^2)(k_r^2 + k_i^2)}$$

$$- \frac{\pi \zeta \beta j_e[k_i - e^{-k_r d}(k_i \cos k_i d + k_r \sin k_i d)]}{ed(1+\zeta^2)(k_r^2 + k_i^2)}, \tag{11}$$

$$\tau_x = -\frac{\pi \zeta \beta j_e}{e(1+\zeta^2)} - \frac{\pi \beta j_e[k_i - e^{-k_r d}(k_i \cos k_i d + k_r \sin k_i d)]}{ed(1+\zeta^2)(k_r^2 + k_i^2)}$$

$$+ \frac{\pi \zeta \beta j_e[k_r - e^{-k_r d}(k_r \cos k_i d - k_i \sin k_i d)]}{ed(1+\zeta^2)(k_r^2 + k_i^2)}. \tag{12}$$

The first terms of Eqs. (11) and (12) are identical to the adiabatic and nonadiabatic torque estimated by Zhang and Li,[7] respectively. These first terms arise from the spatially independent part of the spin accumulation, i.e., the first term of Eq. (7). It should be noted that these terms are independent of the thickness of the domain wall d. For a thick domain wall, these first terms are dominant for spin-transfer torque and the ratio of the magnitude of the adiabatic and nonadiabatic torque, $|\tau_x / \tau_y|$, is given by $\zeta \simeq 10^{-2}$.[7] On the other hand, the second and third terms of Eqs. (11) and (12) arise from the spatial variation in the spin accumulation, i.e., the second term of Eq. (7). As shown in Figs. 1(c) and 1(d), for a thin domain wall, we cannot neglect the spatial variation in the transverse spin accumulation and these second and third terms dominate the spin-transfer torque. It should be noted that these terms are inversely proportional to the thickness d. Thus, for a thin domain wall, the strength of the spin-transfer torque is considerably different from that estimated by Zhang and Li.[7]

Figure 2 shows the strength of the adiabatic torque τ_y and the nonadiabatic torque τ_x renormalized by $\pi \beta j_e / e$ against the thickness of the domain wall d. We denote the torque for $d \leq l_{\text{mfp}}$ by the dotted line because our calculations are restricted for $d \geq l_{\text{mfp}}$; thus, the torques for $d \leq l_{\text{mfp}}$ are not valid. As shown in Fig. 2, for $d \geq 30$ nm, spin-transfer torque is nearly independent of the thickness d. On the other hand, for $d \ll 30$ nm, the strength of the nonadiabatic torque increases as the thickness d decreases. For $d \leq 10$ nm, $|\tau_x / \tau_y| \simeq 10^{-1}$, which is 1 order of magnitude larger than that
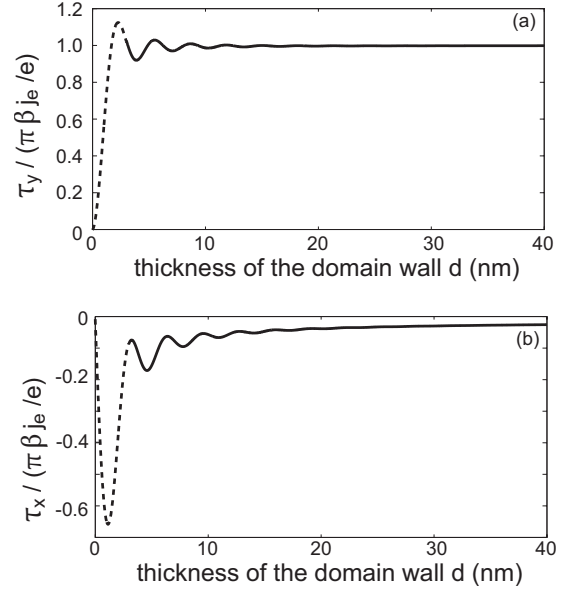

FIG. 2. (a) The strength of the adiabatic torque τ_y renormalized by $\pi \beta j_e / e$ against the thickness of the domain wall is shown. (b) The strength of the nonadiabatic torque τ_x renormalized by $\pi \beta j_e / e$ against the thickness of the domain wall is shown. For $d \leq l_{\text{mfp}} = 3$ nm, the diffusion approximation cannot be applied to the Boltzmann equation, and thus, torque below $d \leq l_{\text{mfp}}$ denoted by the dotted line is not valid.

estimated by Zhang and Li.[7] Moreover, for $d \leq 10$ nm, the spin-transfer torque oscillates against the thickness d with the period of the oscillation given by $2\pi / k_i$.

Let us reveal the parameters which characterize the above behavior of the spin-transfer torque. Assuming that $k_r \simeq \sqrt{3}/(4l_{\text{mfp}}) \ll k_i \simeq \sqrt{3}\omega_J / v_F$ (Ref. 15) and $d \gg l_{\text{mfp}}$, we find that $\tau_y \simeq \pi \beta j_e / e$ and $\tau_x \simeq -\pi \zeta \beta j_e / e - \pi \beta j_e / (edk_i)$, respectively. Thus, for $d \gg l_{\text{mfp}}$, the adiabatic torque is nearly independent of the thickness. On the other hand, for $d \ll 1/(\zeta k_i) \simeq l_{\text{sf}}/(2\sqrt{3}) \simeq 40$ nm, where $l_{\text{sf}} = v_F \tau_{\text{sf}}$ is the spin-flip length, the torque due to the spatial variation in the spin accumulation is dominant for the nonadiabatic torque. For a thin domain wall, the ratio of the adiabatic and nonadiabatic torque is characterized by $v_F / (\sqrt{3}\omega_J d)$, which is the ratio of the precession frequency of the electrons' spin around the localized spin angular momentum due to the sd exchange coupling and the angular velocity of the rotation of the exchange field in the domain wall. For $d = 10$ nm, $v_F / (\sqrt{3}\omega_J d) \simeq 10^{-1}$. The oscillation period is given by $2\pi / k_i \simeq 2\pi v_F / (\sqrt{3}\omega_J) \simeq 2.5$ nm. These estimations can be confirmed by the plots shown in Fig. 2.

When the precession frequency of the electrons' spin around the exchange field, ω_J, is comparable to the angular velocity of the rotation of the exchange field in space, $\pi v_F / d$, the direction of the electrons' spin cannot vary their direction adiabatically, and the nonadiabaticity, which is sometimes called the mistracking effect, plays an important role in the spin-dependent transport phenomena. For example, the terminal velocity of the domain-wall motion is proportional to the ratio of the adiabatic and nonadiabatic torque.[17] The nonadiabaticity is characterized by a dimensionless parameter $\xi = \pi v_F / (2d\omega_J)$.[18] As shown above, the

ratio of the adiabatic and nonadiabatic torques for a thin domain wall, $v_F/(\sqrt{d}\omega_J d)$, is the first order of ξ while the magnetoresistance due to the mistracking effect or spin accumulation is on the second order of ξ.[10,14,15] For conventional ferromagnetic metals with $d \gtrsim l_{mfp}$, ξ is less than unity. Thus, the nonadiabaticity plays an important role in the dynamics of the localized magnetization compared to the magnetoresistance. It should be noted that for a thick domain wall the nonadiabatic torque is characterized by $\zeta = 2/(\omega_J \tau_{sf})$, not ξ, as shown by Zhang and Li.[7]

We compare our results with those of Vanhaverbeke and Viret.[13] In Ref. 13, a time-dependent phenomenological Larmor equation for the magnetic moment in a moving frame is solved numerically and showed that the nonadiabatic torque is shown to be one order of magnitude larger than that estimated by Zhang and Li[7] when the thickness of the domain wall is comparable to the Larmor precession length $\lambda_L = v_F/(2\pi\omega_J)$, which is on the order of a few nanometers. On the other hand, we consider the spin diffusion in the domain wall in a steady state by solving the Boltzmann equation in the rotated frame and analytical expressions of the spin accumulation and spin-transfer torque are obtained. We show that the strength of the nonadiabatic torque increases as the thickness of the domain wall decreases for $d \lesssim l_{sf}/(2\sqrt{3})$. Note that the condition is determined by the spin-flip length l_{sf} instead of the Larmor precession length λ_L. We also find that the strength of the nonadiabatic torque is characterized by the first order of the nonadiabatic parameter $\xi \propto 1/d$. The nonadiabatic torque does not change its sign, as shown in Fig. 5 in Ref. 13.

Since the diffusion approximation is applied to the Boltzmann equation, the present theory is not applicable to the ballistic region $d \lesssim l_{mfp}$. The spin-transfer torque in the ballistic region is obtained by Waintal and Vilet.[19] One can easily confirm that our Eq. (7) reduces to Eq. (12) of Ref. 19 in the limit of $\tilde{T}$, $\tau_{sf} \rightarrow \infty$, where they assume that the local spin-transfer torque is proportional to the spin accumulation. One might expect a simple connection formula between ballistic and diffusive spin-transfer torquelike Wexler's formula for conductance.[20] However, this is beyond the scope of the present Brief Report.

In conclusion, we studied spin-transfer torque in a domain wall by solving the Boltzmann equation for spin accumulation and found their analytical expressions. For a thin domain wall whose thickness is much thinner than the spin-flip length, the ratio of the magnitude of the adiabatic and nonadiabatic torque is about 10^{-1}, which is one order of magnitude larger than that estimated in Ref. 7 and consistent with that in Ref. 13. We also found that the strength of the nonadiabatic torque is inversely proportional to the thickness of the domain wall.

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